Technical Comments

Comment on "Local Nonsimilarity Boundary-Layer Solutions"

M. Coxon* and E. K. Parks†
University of Arizona, Tucson, Ariz.

In a recent paper[1] Sparrow, Quack, and Boerner outline a new method of solution for nonsimilarity boundary layers. In their development, the transformed boundary-layer equation (taking the parameter $\beta = 0$ for simplicity) is,

$$f''' + ff'' = 2\xi[f'(\partial f'/\partial\xi) - f''(\partial f/\partial\xi)] \qquad (1)$$

with appropriate boundary conditions. They point out that the local similarity model is obtained by equating the right hand side of Eq. (1) to zero. For values of ξ which are not small, this is justified by postulating that derivatives of f, f' with respect to ξ are very small.

In proceeding to the local nonsimilarity models, new variables are introduced by writing

$$g(\xi,\eta) = (\partial f/\partial\xi),\ g'(\xi,\eta) = (\partial f'/\partial\xi), \text{ etc.} \qquad (2)$$

Then Eq. (1) and the appropriate boundary conditions are differentiated with respect to ξ to give the subsidiary equation,

$$g''' + fg'' + f''g = 2(f'g' - f''g) + 2\xi(\partial/\partial\xi)(f'g' - f''g) \qquad (3)$$

and its associated boundary conditions.

Now in the development of Ref. 1 Eqs. (1) and (3) are treated as a pair of simultaneous, coupled, ordinary differential equations by assuming that $(\partial/\partial\xi)(f'g' - f''g)$ is small for values of ξ away from zero and consequently the last term in (3) is dropped.

However, the last term in Eq. (3) may be expanded and written as

$$2\xi(g'^2 - g''g) + 2\xi[f'(\partial g'/\partial\xi) - f''(\partial g/\partial\xi)] \qquad (4)$$

Clearly there is no need to drop the first part of Eq. (4) and it is necessary only to assume that derivatives of the g functions with respect to ξ are very small for values of ξ away from zero.

Since the present authors do not have developed programs for solution of these equations, they have not been able to determine what effect retaining the extra terms might have on the accuracy of the two-equation model.

The point made here is of course equally applicable to the development of the three-equation model of Ref. 1.

References

[1] Sparrow, E. M., Quack, H. and Boerner, C. J., "Local Non-Similarity Boundary-Layer Solutions," *AIAA Journal*, Vol. 8, No. 11, Nov. 1970, pp. 1936–1942.

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* Instructor, Aerospace and Mechanical Engineering Dept. Member AIAA.
† Professor, Aerospace and Mechanical Engineering Dept. Associate Fellow AIAA.

Reply by Authors to M. Coxon and E. K. Parks

E. M. Sparrow*
School of Mechanical and Aerospace Engineering, University of Minnesota, Minneapolis, Minn.

The point made by M. Coxon and E. K. Parks was, of course, known to us during the course of the research. It was a deliberate decision on our part to employ the closure condition of the paper rather than the alternative one which they have cited.

In connection with the foregoing remark, it might be well to review the two alternative closure procedures. In one approach, that cited by Coxon and Parks, terms are retained or discarded depending on whether or not they lend themselves to local solutions. In the other approach, that used in the paper, the terms $(f'g' - f''g)$ are discarded in the local similarity model, $\partial/\partial\xi(f'g' - f''g)$ is discarded in the two-equation model, $\partial^2/\partial\xi^2(f'g' - f''g)$ is discarded in the three-equation model, and so forth. That is, in the latter approach, the nth level of approximation is defined by the condition, nth derivative of the nonsimilarity terms with respect to ξ is small. In the view of the authors, the logic of this definition is more attractive than is the pragmatic basis of the alternative closure condition.

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* Professor of Mechanical Engineering.

Errata: "Compressibility Effects on Oscillating Rotor Blades in Hovering Flight"

W. P. Jones and B. M. Rao YES
Texas A&M University, College Station, Texas

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The authors are indebted to Dr. G. A. Pierce for drawing attention to a few typographical errors in the above paper. The corrections needed are: i_r should be i^r in Eq. (42); $\Sigma_{n=0}$ should be $\Sigma_{r=0}$ in Eq. (44); and $2i^r(\pi F/QD)X_0(v)J_0(v)$ should be $2i^r(\pi F/QD)C_oX_o(\nu)J_r(\nu)$ in the last line of Eq. (47).

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